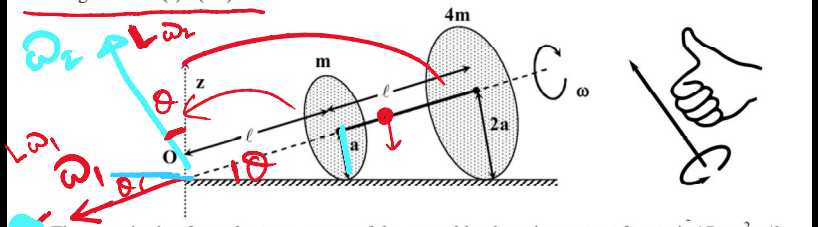
Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $\ell = \sqrt{24}\,a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true?



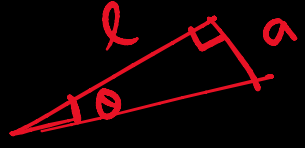
(A) The magnitude of angular momentum of the assembly about its center of mass is $17\,ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\,ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81\,ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$

SOH, CAH, TOA



$\tan\theta = \frac{a}{\ell} = \frac{a}{\sqrt{24}\,a}$

$\sin\theta = \frac{1}{5}$

$\cos\theta = \frac{\sqrt{24}}{5}$

$\tan\theta = \frac{1}{\sqrt{24}}$

A, D

$CM = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{m\ell + 4m\cdot 2\ell}{5m} = \frac{9m\ell}{5m}$

$CM = \frac{9\ell}{5}$

Self rotation

$V = r\omega$

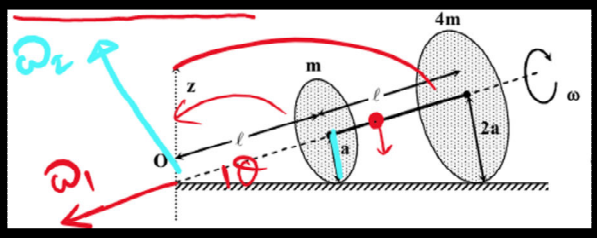
$V = a\omega$

About O axis

$V = r_2\omega_2$

$\omega_2 = \frac{V}{r_2}$

$\omega_2 = \frac{a\omega}{\ell}$



$L = I\omega$

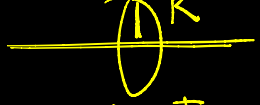
$= I_1\omega + I_2\omega$

$= (I_1 + I_2)\omega$

$= \left(m\frac{a^2}{2} + 4m\cdot\frac{(2a)^2}{2}\right)\omega$

$= \left(ma^2 + 4m\cdot 4a^2\right)\omega$

$I = mr^2$



$I = \frac{mR^2}{2}$

$$= \left(\frac{ma^2 + 4m \cdot 4a^2}{2} \right) \omega$$

$$L_{\omega_1} = \frac{17ma^2\omega}{2}$$

$$L\omega_2 = I\omega_2$$

$$= mr^2\omega_2$$

$$= 5m\left(\frac{9\ell}{5}\right)^2 \cdot \frac{a\omega}{\ell}$$

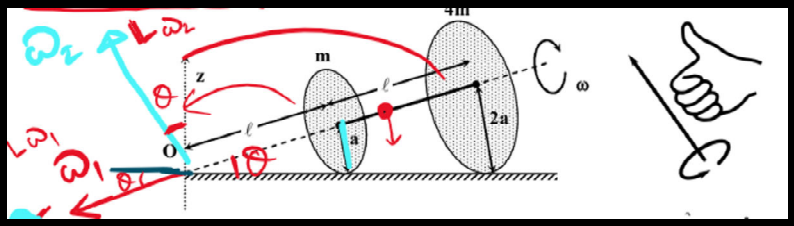
$$= \cancel{5}m \cdot \frac{81\ell^{\cancel{2}}}{\cancel{5}\,\cancel{25}} \cdot \frac{a\omega}{\cancel{\ell}}$$

$$\ell = \sqrt{24}\,a$$

$$= \frac{81\ell a m\omega}{5}$$

$$= \frac{81 \cdot \sqrt{24}\,a \cdot a\,m\omega}{5}$$

$$L_{\omega_2} = \frac{81\sqrt{24}\,ma^2\omega}{5}$$



$$L_z = L\omega_2 \cos\theta - L\omega_1 \sin\theta$$

$$= \frac{81\sqrt{24}\,ma^2\omega}{5} \cdot \frac{\sqrt{24}}{5} - \frac{17ma^2\omega}{2} \cdot \frac{1}{5}$$

$$= \frac{2 \times 81 \times 24\,ma^2\omega - 5 \times 17\,ma^2\omega}{50}$$

$$= \frac{3888\,ma^2\omega - 85ma^2\omega}{50}$$

$$L_z = \frac{3803\,ma^2\omega}{50} = 76.06\,ma^2\omega$$

$$\omega_z = \omega_2 \cos\theta$$

$$= \frac{a\omega}{\ell} \cdot \frac{\sqrt{24}}{5}$$

$$= \frac{a\omega}{\cancel{\sqrt{24}\,a}} \cdot \frac{\cancel{\sqrt{24}}}{5}$$

$$= \frac{\omega}{5}$$